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EDITED BY
W. H. METZLER

ASSOCIATED WITH

EUGENE R. SMITH HARRY D. GAYLORD
GEO. GAILEY CHAMBERS WILLIAM E. BRECKENRIDGE

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SOME HELPS AND HINDRANCES IN TEACHING MATHEMATICS IN THE SECONDARY SCHOOL.*

BY L. E. LYNDE.

It was my good fortune in school and college to come in contact with some exceptionally good teachers and some exceptionally poor ones. The contrasts were so great and the inferences so obvious that I began my work as a teacher in the confident belief that students' failures were due to inefficient teaching. It is needless to add that a decent self-respect accompanied with a little experience soon compelled me to modify my theory and to place far more responsibility on the student than he deserved. I suppose this is the experience of every teacher. With the enthusiasm of youth we tackle our job, confident that we can do it; and when we fail, we distribute the responsibility among ourselves, our pupils, and our subjects. And the responsibility ought to be so distributed, for each has its share.

The subject that we teach is not an easy one to teach or to be taught; nor is it, I believe, a difficult one unless teacher, or student, or both make it so. This is frequently done, and mathematics is proverbially known as a difficult subject. It is a common belief that a few are born with mathematical powers, and that those who are not so blessed can never hope to gain

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them. I don't suppose that many of us believe that. I have never come in contact with a non-mathematical mind except as I have encountered the non-logical mind. So far as my experience goes, a student who can reason in any subject can reason in mathematics; and students who *reason* in mathematics succeed in it.

Of course, it goes without saying that to succeed a student must have the will to succeed as well as the power to do so; and that his course will be the easier if he has a desire to succeed. We all recognize the incentive that interest furnishes, and we strive to make our mathematical work as interesting as possible. We need to remember, however, that interest and achievement go hand in hand in school as well as out of it. Each depends upon the other. Neither goes far without the other. They develop simultaneously. No boy will long maintain an interest in algebra and geometry if he does poorly in them; and no boy will long lack interest in them if he does well in them. No boy *can* do well in mathematics unless he comprehends it. In no other subject is comprehension more vital. In no other subject, I sometimes think, is it less insisted upon. It is here, I am afraid, that we teachers do our poorest work. We so often encourage manipulation in place of thought. We teach our students to depend too much upon their memories and to little upon their reason. We lead them to rely altogether too much upon authority and too little upon themselves. This accounts for many of the failures and for most of the indifference toward mathematics on the part of students.

If we demand little but manipulation from them, it is unlikely that they will see anything else in mathematics. The figures, the equations, the demonstrations that a boy writes should be the expression of his own thoughts—thoughts that, if not his originally, have become his by possession. The need for this in geometry is obvious and is recognized. I believe it is even more necessary in algebra. Too many a boy gets whatever thought he has from his own written statement, *after he has written it.* What he writes is not a record of his thought. If he pauses to contemplate his written statement, he may get a thought from it. Many do not pause; few contemplate. Parrot work, whether it be in solving a problem, in demon-

strating a theorem, or in solving an equation ought to be frowned on to the limit. This is particularly true in solving algebraic equations.

Many a boy solves algebraic equations by carefully following carefully stated rules with the accuracy with which an accountant adds numbers on an adding machine, and with no more comprehension of his processes. The fault I have to find is not that he does it, but that he is taught to do it. It is not his fault that he is prevented from knowing algebra. The successive eqution that a student writes in solving a given equation should be the conscious expression of new number relations, thought out by himself. They should be the visual expression of his own thoughts,—thoughts that he thinks first, and writes afterward. If they are not, then he is not using algebra, as I understand it; and in my judgment, his teacher is at fault.

How often one meets the following:

$$\frac{3x-7}{x^2-5x+6}=\frac{1}{x-3}+\frac{1}{x-2}=$$

$$3x-7=x-2+x-3.$$

The extra equality sign isn't a careless slip—that is, it usually isn't. It usually betrays utter confusion of thought, or no thought whatever. Such an occurrence provides a text for a thorough sermon, which I think a teacher ought to preach, on what an algebraic equation is and on what it is to solve one. I frequently find it necessary to remark that an equation is not a pair of scales (some textbooks to the contrary notwithstanding), and that it need not be two numbers with a pair of short horizontal lines between them. I find that it helps some boys to comprehend if I remark that "six and two make eight" is as much an equation as "$6+2=8$." The word "equals" has so many uses and senses that a definite definition of it in algebra is helpful, even to a beginner. I like to define it as meaning "represents the same number as." This definition gives clearness to the idea of a root of an equation, and emphasizes the fact that each member of the equation is a function—an idea quite essential, to a real comprehension of the solution of an equation.

But if the student writes:

$$\frac{3x-7}{x^2-5x+6}=\frac{1}{x-3}+\frac{1}{x=2},$$

$$3x-7=x-2+x-3,$$

$$x=2,$$

how often he writes totally unconscious that each successive equation he has written states a deduction from the preceding equation,—thinking perhaps that he has merely changed the dress of his original remark, and like as not led to think so by the phraseology of his text or teacher. How misleading, for example, is the statement that "adding the same number to both members of an equation does not destroy the equality." Would not the student get a better grasp of the facts if he were told that adding the same number to each member of an equation does destroy the old equality and gives a new one. And how often we mathematicians disguise a process by saying "or" when we mean "therefore," *e.g.*:

$$2x=6$$

$$\text{or} \quad x=3,$$

or by saying "and" when we mean "or" *e.g.*, in

$$x^2-5x+6=0, \quad x=2 \text{ and } 3.$$

These seem like petty criticisms, but I believe they account for many misconceptions on the part of students.

I know it is the fashion in some quarters to laugh at the old axioms about "adding equals to equals" and "multiplying equals by equals." As specimens of English these axioms may be open to derision, but as dynamic principles for aggressive action on the student's own initiative, they have not been replaced by any modern notion with which I am familiar. It is at least true that using them students can transform equations intelligently.

If a student has solved equations mechanically, I know of nothing which helps him more to get his bearings than such simple questions as the following: If $3x+2=8$, how much is $3x+5$? It is surprising how that little question will perplex

him. Had the question been "how much is $3x$," he would have answered without hesitation, for the transposing of that 2 is the established order of procedure. But the idea of wishing to get the value of $3x+5$! Eventually he sees that since $3x+5$ is 3 more than $3x+2$, $3x+5$ must be 11. (Of course some ingenious boy solves the equation for x and then computes $3x+2$. That adds emphasis to the point of the question.) When this has been answered, I ask: If $3x+2=8$, how much is $6x+4$? $6x+5$? $9x^2+12x+4$? $\sqrt[3]{3x+2}$? etc. Such questions bring out the relations which exist between one equation and a following one in a solution. They give a student some sense of what the following equation should be; but, best of all, they give him an idea which some students miss, and which a student ought to get early in his work, that the following equation may be *any* number relation he chooses to deduce from a given euqation. In this deduction, his first care should be whether the deduction is true, and his second care whether it serves his purpose. The axioms are his guide in the former and experience his guide in the latter.

I frequently ask a class to write out a solution thus:

If $$\frac{3x-7}{x^2-5x+6}=\frac{1}{x-3}+\frac{1}{x-2},$$

then $$3x-7=x-2+x-3,$$

then $$3x-7=2x-5,$$

then $$x=2.$$

This scheme appeals especially to those who have studied geometry. They recognize the logic at once. They realize that at any stage of the work, the next equation to be written is the equation which expresses their own deduction at the moment, and best of all, they realize—what so often isn't appreciated—the fact that the ordinary solution of an equation isn't a solution at all, but is the converse of a solution. The checking of an equation becomes to them more than a means of detecting errors; it is an integral part of the solution. (The work above doesn't show that 2 is a root. It shows that if the equation has

a root, that root is 2. The check determines whether or not 2 is a root of the equation.)

I hope I am not misunderstood. The solution of equations may be taught purely as an art. A set of rules may be drawn up by following which one may determine the roots of an equation without checking. I am not attempting to argue that students can't be taught to solve equations in that way. I believe and I am trying to give utterance to the belief that this is a better way. (Of course 2 isn't a root of the above equation. One can explain away the difficulty in this example by showing how a root may be introduced into an equation by multiplying by an expression which involves the unknown, and moralizing therefrom. That is the mechanical way of meeting the situation. That to my mind is an additional reason why the deductive method of solving an equation is preferable.)

I believe that our aim in teaching mathematics should be to develop the mathematical sense of the student. The art of computation is then merely this sense applied. The mere doing of examples will, of course, enable a student, if he does enough of them, to do others like them, but it prepares him very poorly to do others of a different type. A student must know how to perform the six fundamental operations on the numbers of algebra. This requires drill, but not mere drill. The quality of the examples done counts far more than the quantity. Many texts have too many examples which students can do and do do by rote. In doing them the student applies no principle and seeks no goal. He does each like the last. His question is not "what am I seeking to do, and how shall I do it?" but "what shall I do next?" And all too frequently that question—"what shall I do next?"—is answered. I believe we give too much attention to the steps our students take and too little instruction as to where they are going. We teach them how to add fractions, but we say little or nothing as to what is meant by the sum of two fractions. We teach them how to multiply radicals but not what the product of two radicals is. We teach them to factor as though factoring were the sole purpose of algebra, but do we teach them when to factor and why? It takes some weeks each fall for me to convince some of the boys who come to me that $(a+b)(a-b)$ is not a better answer to all ques-

tions than $a^2 - b^2$. We teach students to reduce fractions by cancelling common factors in numerator and denominator, and the trouble is that is literally what they do. I am not giving utterance now to the old complaint that they cancel quantities that are not factors. I am calling attention to the fact that many students persist in writing

$$\frac{6a - 2d}{4b - 18d} = \frac{\cancel{2}(3a - d)}{\cancel{2}(2b - 9d)}$$

instead of dividing numerator and denominator by 2. Some never reach the stage where they reduce

$$\frac{x^3 - y^3}{x^2 - y^2}$$

without writing

$$\frac{\cancel{(x - y)}(x^2 + xy + y^2)}{\cancel{(x - y)}(x + y)}$$

Students who express themselves in that way probably haven't a very clear idea of what they are doing. It is usually another case of being so bound up in the mechanics of an operation, that the operation itself is lost sight of.

Many a student knows that $\sqrt{8} = 2\sqrt{2}$ just as he knows that 2 times 3 is 6. He has memorized the fact. He can't simplify $\sqrt[5]{64}$ because that number wasn't mentioned in the book he studied. The same boy wouldn't hesitate a moment to write $\sqrt{4 + a} = 2 + \sqrt{a}$, and there isn't the slightest reason why he should hesitate, for he used no principle in simplifying $\sqrt{8}$, and probably doesn't use principles in algebra. It may be that his introduction to the reduction of surds was an explicit as the following illustrative example given in a very good textbook:

$$\sqrt{\tfrac{2}{3}} = \sqrt{\tfrac{6}{9}} = \sqrt{\tfrac{1}{9} \cdot 6} = \tfrac{1}{3}\sqrt{6}.$$

This reveals essential steps in the process, but the only principle of surds involved in the work—in passing from the third to the fourth expression—is omitted. The one step that needs the greatest elucidation gets none. Is it any wonder that a student often gets only a fact and infers no principle?

Rules are at times a great convenience, and at times almost a necessity. But I wish that authors of texts did not feel

obligated to state a rule for every process. How often a rule is stated when it should have been omitted, and how often a desirable rule is stated just when it shouldn't have been stated. An author has given a discussion, say, or has performed several illustrative examples. The class is on the point of grasping what he has done. It is ready to infer from the work done a principle for future action. It needs the experience of phrasing its own thoughts and testing them. The whole process is spoiled by the foresight of the author. He states a rule, in type so bold that the normal student sees nothing else on the page. At any rate he reads it first, and is prevented from using his own powers of inference. I can illustrate this by an instance in an exceptionally good textbook in trigonometry. The author gives an unusually complete discussion of interpolation—clear enough for any mind to comprehend. But instead of giving the student a chance to test his comprehension, the author spoils his discussion by framing a rule. He tells the student that to find a function of a given angle, he should write down the function of the next smaller angle in the table. The author then tells him how to find the correction, and adds this significant clause: "if sine or tangent, add the correction; if cosine or cotangent, subtract it." Such procedure seems to me almost criminal. If a boy has secured any comprehension of interpolation from the discussion, he doesn't need that rule. A glance at his tables will tell him whether the function is increasing or diminishing. If he does need that clause, he ought to be forced to study the discussion again, and to get a comprehension of it. But he isn't compelled to; he is told what to do, and he does it. He gets answers to four places correctly and thinks he knows all that is practical in trigonometry, and is encouraged to think so. Had he not been told what to do, he would have discovered it. He would have experienced the thrill and satisfaction of a discoverer. He would have been better equipped for future discoveries. He would have gotten more courage for the next venture and more faith in his own powers.

In solving equations by factoring, what more does a student need than appreciation of the fact that the product of several numbers can't be zero, unless one of the numbers is zero! But how many books stop there? Most of them add a rule—

giving to it as much prominence, if not more, than they give to the principle—and saying in some form that one should "*set*" each factor equal to zero. This implication of arbitrariness isn't lost on some minds. If one sets each factor equal to zero, why not set each equal to 10? Some do. I can't see that a rule here serves any useful purpose. Most students would know more mathematics if our texts contained fewer rules.

I referred some minutes ago to the increased reverence for the checking of an equation that a student has, if he comprehends the successive equations he writes in the so-called solution of the equation. There are more students than there were some years ago who realize that the letters used in algebra represent numbers. The modern insistence on the use of the check is responsible for this, I am sure. But I believe that the check has not contributed as much as it might toward increased knowledge and power in mathematics, because of the careless way in which we allow our students to check their results. I know many texts and many teachers who not only allow, but teach their students to check an equation by substituting throughout the equation the apparent value of the unknown for the unknown, and operating on the result as though "solving an equation." In checking

$$\frac{x^2}{2x^3-2}=\frac{2x}{3x^2+3x+3}-\frac{1}{6x-6}$$

for $x=-\frac{1}{5}$, they write: Check

$$\frac{\frac{1}{25}}{-\frac{2}{125}-2}=\frac{-\frac{2}{5}}{\frac{3}{25}-\frac{3}{5}+3}-\frac{1}{-\frac{6}{5}-6}$$

$$-\frac{5}{252}=-\frac{10}{63}+\frac{5}{36}$$

$$-5=-40+35$$

$$-5=-5.$$

This form of check may do for the expert mathematician, who knows more about indentities and reversible operations than students who are studying elementary mathematics know. It may do for him, I say, provided he is doing it as scratch work for his own use, and is making no attempt to state his thought

and conclusions formally. It is almost worse than no check at all for the boy who is learning algebra. It teaches him to assume the one thing he is trying to find out—whether for this value of x the left member and the right member represent the same number. It teaches him to play with the equality sign and not to use it to make a serious statement. But worse than this, it leads many students to infer that a statement can be established by showing that truth flows from it. The first statement in the check is true, he infers, because from it he obtained a conclusion obviously true, *i.e.*, $-5=-5$. I believe that we can teach nothing more practical in mathematics than that no statement can be proved by assuming it. Obvious as that truth appears, a goodly portion of the human race needs instruction upon it. I wish that one qualification required of every voter was evidence of his appreciation of the fact that the converse of an established proposition is not necessarily true. Of course I realize that this requirement cannot be made. It would disfranchise too many voters. But I am almost ready to say that the entire time ordinarily devoted to mathematics is justified, if our students emerge from it fully realizing that the converse of an established proposition is not necessarily true and that a proposition can never be established by assuming it. Insistence on some such arrangement of the check as follows, greatly assists the student to work understandingly:

When $x=-\frac{1}{5}$,

$$\text{L. M. (left member)} = \frac{\frac{1}{2\frac{1}{5}}}{-\frac{2}{1\frac{2}{5}}-2} = -2\frac{5}{52},$$

$$\text{R. M.} = \frac{-\frac{2}{5}}{\frac{3}{2\frac{1}{5}}-\frac{3}{5}+3} - \frac{1}{-\frac{6}{5}-6} = -\frac{10}{63}+\frac{5}{36} = -2\frac{5}{52}.$$

Frequently a check is so written that it doesn't involve as much bad reasoning as the earlier check. *E.g.*,

$$4+(x-3)(x-5)=15-(7-x)(2+x)$$

$$x=6.$$

Check.

$$4+(6-3)(6-5)=15-(7-6)(2+6)$$

$$4+3=15-8$$

$$7=7.$$

The fundamental error is as bad in this as in the earlier one, and in my opinion no check in this form should be accepted.

Another instance of the same bad habit occurs repeatedly in trigonometry, in establishing some of the so-called identities. To take a simple illustration:

$$\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$$

$$\sin^2 x = 1-\cos^2 x,$$

$$\sin^2 x + \cos^2 x = 1,$$

$$1 = 1. \qquad \text{Q.E.D.}$$

How often this is allowed to stand as a proof not that $1=1$, but that

$$\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}.$$

So long as mathematicians accept such proofs as this, I shall not initiate a referendum petition for a change in the qualifications of a voter.

It sometimes happens that students who do fairly well in abstract algebra have little success in solving problems. I find that this is sometimes due to the mistaken point of view from which they approach the problem. When a student says "I can't get an equation," I encourage him to keep trying. I assume that he knows what he is trying to do, and that he is likely soon to do it. But if he says "I can't get *the* equation," I recognize a job for myself. That boy, if his speech reveals his mind, looks upon a problem as a puzzle, in which some one has hidden an equation which it is his duty to find. He sets out looking for that equation; and if he has been unsuccessful in the past in playing this kind of hide and seek, he sets out with little expectation of finding the equation. It is some comfort and inspiration to him to be told that a problem can be solved by more than one equation; that in most problems there are many different number relations; that what he should seek is not a particular equation, but any equation involving his unknown, and not necessarily the equation in the mind of the author. This gives him hope and some courage. But if he

still seeks for an equation *as such*, he is unlikely to find it. He needs to be told to think out his number relations in English, and when he has obtained an equation in English to translate it into algebra.

The equations used in solving problems necessarily deal with concrete rather than abstract numbers. I have found that a requirement that a student shall indicate the denomination of the number he has in mind in his equations is of great aid to me in interpreting his work and to him in clear thinking. *E.g.*,

$$\frac{3}{x}\text{ hrs.} - \frac{2}{x+3}\text{ hrs.} = 3\tfrac{1}{2}\text{ min.}$$

This is particularly illuminting if the conditions of the problem are such that the 3 represents miles and the x speed in miles an hour. Not the least advantage is that many half-baked ideas can be discovered in this way and put back for further cooking. For example, in the typical work-problem equation

$$\frac{1}{x} + \frac{1}{y} = \tfrac{1}{3},$$

where A and B together do a job in 3 days, the boy who doesn't comprehend is pretty sure to write:

$$\frac{1}{x}\text{ days} + \frac{1}{y}\text{ days} = \tfrac{1}{3}\text{ days},$$

or some other statement which reveals his careless thinking as the abstract equation

$$\frac{1}{x} + \frac{1}{y} = \tfrac{1}{3}$$

would not do.

Another detail which I believe is very important in its practical bearing on the solution of problems is the insistence that a student shall definitely define his unknown; that is, that he shall write not "let x = the cost," but "let x = the cost in dollars," or beads, or cents, as the case may be; not "let y = the speed," but "let y = the speed in yards a year." Experience proves that few students can afford to be less explicit, and that many profit by this attention to detail.

If then our chief concern in teaching mathematics should be, as I believe, to impart a knowledge of its principles, and a comprehension of them, it follows that the tests and examinations we give our classes should seldom, if ever, call for mere duplication of work already done. Examination questions should very enough from questions already drilled upon, to ensure that the student has gotten hold of the principle involved and can use it. Students, like the rest of the human race, do little more than is expected of them. A class is more alert, more eager to catch a truth, and the whole of the truth, if it realizes that a question will soon be asked which requires for an answer more than the repetition of what text and teacher have said. A good examination question never departs widely from questions already discussed in class, but it should seldom be identical with any of them.

This ideal is easily obtained in algebra and trigonometry. It must be modified frequently in geometry. But even here our classes can and should do better work. It is possible to insist on actual demonstration by the pupil and to get it. The demonstration of a theorem need not be a mere recitation. No student should receive a pass grade who merely recites geometry, and in my judgment, few will be candidates for a pass grade on that basis, if the ideal of grasp of principle and application of it are duly presented. I question the effectiveness of tests in geometry containing, say, three book theorems for demonstration and one question on so-called original work. Other things being equal, the ratio ought to be reversed. (Of course I do not mean to assert that a test of the first type should never be given. I give them at times myself. I am speaking of the undesirability of confining examinations to that type, or of making them predominantly of that type.) Certainly no student should be passed in geometry if he has so failed to catch its spirit and purpose, that he cannot use geometry.

Apparently we teachers of geometry do not insist that our pupils shall know enough of the facts of geometry. Every student ought to know as a fact each theorem which has been deemed important enough to be placed as a proposition in the textbook. He should know it as a fact even if he cannot prove it, and it should be at his tongue's end for statement. And he

should know scores of other facts which have occurred as exercises. It has been my experience that students' inability to do so-called original work in geometry is more often due to ignorance of facts than to inability to use them. Numerical problems based on the facts of geometry not only furnish material for coördination between algebra and geometry, but they offer one of the best means for drilling and testing in geometric facts. The average text in geometry could well contain a larger number of numerical exercises. And I would place them sufficiently distant from the theorems upon which their solution depends, to permit a student to use his own initiative and judgment.

No teacher ought to be satisfied with his own work, and I can't recall that I ever met one who was. We are all eager to perfect our work by improved methods of teaching, the introduction of new material, and the elimination of old material when we are sure that such changes operate for the real benefit of the student. Some time ago the complaint arose that students in the sciences were not able to solve the equations they met there, and we teachers of mathematics were blamed for this situation, and we deserved it. Into the new texts in algebra the equations of physics and other sciences were placed; and we have drilled our students on these equations until the complaint of the science teachers isn't as loud as it was. But I wonder if we thereby have made our teaching of algebra more practical. Unless by this means we have improved our teaching of the solution of algebraic equations as such, I don't believe we have done so. A student who knows how to solve equations will solve them, whether he meets them in physics or in biology, in June or in August, whether they are expressed with x, y, and z or with M, F, and T. It is a mistake to think that algebra is made more practical by mere drill on any particular application of algebra. Some believe that time spent in teaching complex fractions, the binomial theorem, the square roots of binomial surds, etc., is wasted. It is doubtless true that some of us spend more time on these topics than we should, and that the occasions for direct application of them are few. But it by no means follows that they should be entirely omitted from our courses. Shall we teach complex fractions? The answer

to that question should not depend on whether a plumber, or an engineer, a banker, or a statistician uses complex fractions. The real question before a real teacher is, does the study of complex fractions assist a student enough in getting a grasp of mathematical principles to make the study of complex fractions worth while.

There are many committees working at present to improve the teaching of mathematics. Much has been done and much will be done in elimination of work here and the introduction of new work there, in the rearrangement of material and of courses. We look for a greater diffusion of mathematical knowledge as a result of the labors of these committees; but I believe there is a greater need than this inspecting and auditing, this rejecting and renewing of material. There is, I believe, a vast difference between teaching a boy how to do mathematics, and teaching him how to do it himself. We may add special topics to our courses, or omit them; we may drill in this spot or in that; we may teach algebra one year, and geometry the next; we may teach algebra on Mondays and geometry on Tuesdays; we may teach both together on Wednesdays; we may use graphs to illustrate simultaneous equations, or simultaneous equations to illustrate graphs; we may drill our would-be engineers on engineering problems, and our would-be chemists on chemical problems—unless we teach our students to comprehend their processes, they will only be able to do things like those they have done; they will not be able to do others. They will not be able to apply mathematics. There is no substitute for reason in a subject so essentially rational as mathematics.

PHILLIPS ACADEMY,
ANDOVER, MASS.

THE EFFECT OF POST-ARMISTICE CONDITIONS ON MATHEMATICAL COURSES AND METHODS.

By Harry English.

Events are happening so rapidly that one fancies he is looking at a quick-moving kaliedoscope of topsy-turvey happenings colored by the ever-changing play of varied human emotions and interests throughout the world.

At the time the heading on the program was suggested it seemed that there was a reasonably fair chance that conditions as then existent would crystallize rapidly and become stabilized to such an extent as to affect vitally and basically our theory and administration of education. In many ways this seemed to be necessary and desirable; in other ways, most undemocratic and revolutionary.

I had prepared a more or less satisfactory summation of these conditions, and had attempted to deduce some possible conclusions as to their immediate effect and their more remote bearing on our courses and methods of teaching. I had wandered through the maze of offices in Washington where trained investigators were trying to solve the many problems connected with education of all kinds as affected by war conditions,—when the armistice was signed. At once the vital efforts connected with the present draft ceased; the camps of drafted soldiers began to dissolve; the college students' training camps to disintegrate; the war workers to "homeward wend their weary way"; the trained specialists to "fold their tents like the Arab and as silently steal away" and "leave the world to darkness and to me." I had a useless paper on a subject which might have been of interest yesterday, but which, to-day so far as life is concerned, is in a class with similar curiosities found on rolls of papyrus long buried with the Egyptian dead.

Under the circumstances, I most gladly welcomed a suggestion from our President, Dr. Hawkes, to direct my remarks along the lines of the effect of post-armistice conditions on our

courses and methods. In the chaos into which I was plunged, I felt as competent to speak on one subject as another, so I made a palimpsest of my original paper, and sailed madly into an uncharted sea, with but a few hours at my disposal for preparation and practically no data at my disposal. I have never posed as a prophet so the rôle is a new one. It is to be noted however that both England and France long ago began a comprehensive study of conditions of all kinds—industrial, commercial, financial, economic, educational—looking towards a complete readjustment wherever made necessary or desirable because of the war. They are about to set this machinery in motion.

As yet our own country has done nothing as a country, though some individual agencies are at work. I have a feeling that something national should have been done some time ago. At present we are drifting especially as to educational reconstruction. One or two states have given the matter some attention, but it is a significant fact that labor is about to appoint general committees, one of which is to make sweeping, comprehensive suggestions as to a plan of education applicable to the entire country, and in this they are to have expert help.

It is a pity that the war educational agencies did not carry on. It is almost criminal that regular educational associations and organizations do not immediately bestir themselves.

Let me very briefly allude to factors during the past year or two which have changed markedly the attitude and receptivity of our people in some matters.

A. Before our Declaration of War.

I. The awakening of the various peoples within the confines of our own country to the true meaning of loyalty to our own country, and the creation of a real national sentiment and of a real American people with national ideals, unity of purpose and will to accomplish.

II. The creation of unified agencies on a vast scale to aid the stricken peoples of invaded Europe, accompanied by a real personal interest on the part of millions of our own people in the people helped as real flesh and blood persons and not as abstract objects of charity.

III. The assertion of a national conscience akin to that of an individual conscience in dealing with others.

IV. The implanting in the heart of every American boy and girl, and man and woman, of the absolute dependence of the government upon the individual ability and willingness to do to the utmost in a democratic form of government all things necessary to vindicate and re-establish martyred and despoiled peoples.

Many other conditions naturally suggest themselves.

B. After our Declaration of War.

I. A prompt and cheerful acquiescence on the part of the people, to the drastic provisions of the first draft—a first necessity to win the war.

II. The equally as prompt and cheerful acquiescence of the people, corporations and other big business entities, to the demands of the government as to the control and taking over of railroads, industrial plants, etc.; the limitations as to: food, coal, duration of the business day, general freedom of action, and in fact anything and everything however dear to the heart of the liberty-loving American people—in order to win the war.

III. The overpowering response to the various Liberty Loans by an ever increasing number of subscribers, with the universal, tremendous, intensified American patriotism shown in the grand rallies held for the first loans, and the stern tenseness and determination of the people to see the last one through in spite of the deadly epidemic which raged.

IV. The habits of thrift and economy formed; the real understanding of economics and economies; the concentrated work of all kinds done by the school girls and boys, the collegians, the men and women of all classes and in all pursuits to see our own Army and Navy through, while still carrying on the charitable work of relief for the increasing millions of dependents in Europe; the intense interest in our own soldier and sailor boys, individually and collectively.

V. The levy of enormous taxes, especially the income tax.

VI. The tremendous increase in our wealth and in the cost of living.

VII. The wonderful attitude of our people as regards the second and last draft law, invading as it did every activity in the country and tending to disorganize every institution, every agency, every business and every home in our country which was founded upon a deadly antagonism to such arbitrary power to which however it yielded for the time being to bring to those people of the world deserving and desiring it, a permanent freedom such as we have heretofore enjoyed.

All of the foregoing leads naturally to a brief consideration of some of the effects of this law upon education and the educational institutions of our country, and has been presented as a background necessary to the proper understanding of the general effect of the lowering of the draft age for the second draft, and as indicating in a most cursory way some of the conditions back into which we were abruptly thrust when the armistice was signed, November 11, 1918. A mere glimpse of the condition will be given.

The Period during which the Second Draft was Effective.

Much literature of a general as well as specific nature with reference to war activities appeared during the summer and early fall of 1918.

Among the authoritative issues has been a series of pamphlets issued by the Bureau of Education entitled "School Life." Seven pamphlets have been issued so far, the first on August 1, 1918; the last so far on November 1, 1918. The articles deal almost exclusively with problems affecting the reconstruction of courses and the work in educational institutions to meet the requirements of war conditions.

Another important work is that discussed in issues of the *American Mathematical Monthly*, September, October, November, 1918. Still another, as yet in the making, that of Professor Young of Dartmouth College.

Whether Congress intended it or not, the result of lowering the draft age to eighteen years was to make every college in

its entirety a war camp so far as boys and men were concerned. Had the draft age been set at nineteen or twenty years, roughly speaking about half of each college would have been under the old regime, and half under the new. As it was, to all intents and purposes the Secretary of War became the Director of Education for all male colleges, with a divided authority in co-educational institutions and with no authority except by force of example in female colleges.

To a large extent members of the fourth-year class in all our high schools were similarly affected, and also individuals here and there in the third-year class.

The War Department called to its aid trained specialists and began to formulate plans as to administration and as to setting into motion of intensive composite courses in colleges. Many college professors promptly began to consider the subject matter of courses and methods of presentation.

Special emphasis was laid upon the necessity for intensive work in industrial courses in high schools and a call was made for intensive work in all branches taken by boys of eighteen years or over with the view of having these boys graduate into the college camps every twelve weeks instead of every eighteen weeks.

It was urged at first that care be exercised in selecting properly trained teachers, especially manual training teachers. This proved impossible, as it turned out, because the industries and the draft took away the bulk of the latter class of teachers and left no visible supply upon which to draw.

In fact the teachers shortage problem became a tremendous problem, and a migration of teachers took place in order to find salaries upon which to live. The teachers formed the one great class of war workers whose salaries, authorities were not forced to raise.

The intensive work in colleges necessarily modified the character and the amount of academic training. The effect upon secondary schools seemed merely the elimination of non-essentials if anywhere found, and the lengthening of the recitation periods for those pupils affected. In our own high schools in Washington, which are co-educational, many students, not affected by the draft, both boys and girls, desired to take ad

vantage of these courses in order to shorten their time at school; heads of departments prepared suggestions for modified courses which contained all essentials, while improved methods of teaching were put in operation by trained teachers selected from the regular corps. Some classes began at 8:30 A.M. instead of at 9 A.M. It was hoped that habits thus inculcated in these pupils, by force of example would extend throughout the entire school and that we might return to the basis of regular work in vogue some twenty-five years ago for all classes, but which had just been provided as special intensive work for the few. The influenza epidemic closed the schools for a month, but after reopening, allowing time for proper adjustment, redoubled efforts were made to accomplish the work desired. It was hoped that we could get information from the colleges as to their work so that we could get in line if possible to give direct help.

The armistice was signed, and in an inconceivably short time pupils began to withdraw from intensive classes which finally disappeared. The buoyancy and mercurial disposition of youth reasserted themselves in the girl and the boy who for several years had been in the atmosphere of national endeavor doing a woman's and a man's work. The spur and whip of stern necessity had ceased to operate and old habits began to function. It may be that this spirit of itself will be the salvation of the people to enable them to forget speedily the horrors of the present, and that the real helpful effects will emerge later and be shown by real work and real appreciation of what the present and the future really mean. Longfellow, the poet of childhood, may be, after all, a real prophet. The present and future are fraught with danger.

For a time it seemed that the college requirements of last year would be swept away, and that induction into college would be largely governed by a war policy. It was asserted by some, very erroneously, that the junior high school would pass into the discard, by others that national supervision or control of education with the military instruction strongly predominant would ensue; by others that education would be industrialized. It is not believed that such will be the case.

There is a very real fear of a somewhat minor nature in

secondary schools, owing to changes in certain secondary courses made practically at the behest of the War Department, especially as regards industrial work, that the colleges may not make due allowance therefor, and may unintentionally work hardship as regards those desiring to enter college, by insisting upon the exact credit values in the subjects which were required by colleges before the taking over of the colleges by the War Department and before the lowering of the draft age so vitally affected secondary schools. I am sure that every effort will be made by both to meet the successive dislocations in the fairest possible spirit.

Especially will this be necessary when we recognize that all over the country, secondary schools and colleges have been almost stripped of teachers because of the draft or other necessary war work, and that a period of real readjustment is necessary.

Whether all of these teachers will return is a problem. There are so many opportunities for personal betterment elsewhere that the temptation to abandon the teaching profession which has small salary recognition is well nigh irresistible. If the teachers do return, however, they will bring back to the classroom a vigor and breadth of view because of actual contact with realities which will do much to secure that so-called practical method of treatment which has been so long and so loudly demanded. I am not sure that we do not have it, and that critics have not taken the trouble to discover it. The habit of criticism may be so firmly fixed as not to be eradicated, no matter what the conditions or the truth may be.

Let us recognize further that there are in our schools and colleges more or less firmly implanted, various organized forms of voluntary work closely related to war activities of a charitable or helpful nature which probably will continue for years to come both at home and abroad, together with all that background of other activities briefly characterized at the beginning of this paper. Pupils and teachers for some years have formed part and still are part of it. The supreme moving cause seems to be suddenly removed. Some of these conditions may disappear along with the intensive work. Some however have crystallized into habits which must be considered in any scheme

of readjustment of courses. Some reformers urge that they be made regular courses.

Many of our college boys and boys of college age have been in the midst of most vital realities. If they return to college they will have the attitude of experienced men of the world and this cannot be ignored in classroom teaching. To a certain extent this is true of our girls. Indeed many young children have received the pay of men and women and have mingled with men and women as more or less equals. How will they act when they return to the school room?

On the other hand, an incredible amount of illiteracy has been disclosed in army camps, and certain steps should be taken to eradicate it, especially as vast numbers of such men seem so eager to learn who will shortly return to civil life. Of course I do not mean that these men lacked natural intelligence, for in a great number of cases this was of a high order.

Also we must recognize that a tremendous impetus has been given to all trades and forms of labor, and that distinctive schools with such training in view, or the engrafting of such courses upon courses now established or the introduction of such courses into schools now existing must surely follow in increasing numbers. The direct monetary returns therefrom are very great. It is significant that labor is setting in motion with expert help, a definite comprehensive plan of education.

It is also to be noted that the National Education Association and the Association of College Presidents are back of a bill to create a Department of Education which is to take over the functions of such emergency commissions and educational activities created by Congress, as the President may direct. It is also to have certain specified functions of its own. This seems to be in line with the general tendency towards governmental control of all agencies necessary to win the war.

Whether this same spirit will be as intense, now the armistice has been signed, remains to be seen. There seems to be a tendency now to de-centralization and a reversion generally to pre-war conditions.

How far a determination of courses and methods used, as well as of subjects taught, will be affected by these tendencies is uncertain. Undoubtedly there will be a prolonged struggle if indications mean anything.

So far the considerations have been general, it now seems advisable to come down to particulars.

As to Secondary Schools.

Practically nothing has been done. Action was scarcely begun before it was stopped. In all my interviews with officers both before and after the armistice was signed there seemed to be nothing in sight for secondary schools other than practical applications of trigonometry or plane geometry such as obtaining heights, distances, etc., or working out the elementary problems of navigation. It was urged that surveying instruments be provided and that the boy become automatically familiar with them, that practical things be kept in view, that elementary computations be made quickly and without paper, that the boy with a real liking for mathematics and with a quick alert mind was far more desirable than one with a head full of knowledge, but slow in application.

A man to meet emergencies was the ideal. This of course was evident so far as this war was concerned to ensure real participation by the American army because of the time element; for navigators, aviators, and officers connected with more or less mobile artillery would need such characteristics and much more.

It was a saying that men should be taught to shoot in their bathing suits, *i.e.*, if every material practical device and aid were suddenly swept away, the men with their knowledge of formulæ could calculate distances, trajectories, etc. This surely was the case at Chateau Thierry, and if men ever shot in their bathing suits it was there, for all telephone communication with the retreating front had been destroyed, all observation posts, balloons and aeroplanes to give distances and other information were disappearing. The artillery had to abandon their ready-made methods of firing and had to calculate mainly in the old-fashioned, long-hand way to turn the retreat into a stand and then into an advance. The traditions of the army and of mathematics were maintained and lived up to.

As a result of the war there has come into being a series of war problems which met the test of springing up from one's environment so far as men and boys were concerned, but

which were of scarcely any interest to girls, who seemed to be sadly neglected so far as adaptations of mathematics were concerned because of the war. It was left for them to obtain human interest from the mathematical courses as best they could. And yet, even now the war problems have begun to recede and before long may pass into the discard together with problems of a more remote antiquity.

It is quite possible indeed that they will be the antiquities of tomorrow.

In this connection I am reminded that thirty years ago and for years thereafter in our Washington high schools before the springtime became the play time for athletic teams and the rest of the admiring pupils, and before there were so many other outside diversions, pupils learned more trigonometry and surveying in the same time that is now devoted to trigonometry alone. We had the very thing then that the army officers say we should have now.

There were certain problems in surveying, using the compass, the sextant, the level, the transit and the plane table.

The problems were as follows: (*a*) distances between inaccessible objects and heights of accessible and inaccessible objects: (*b*) survey for area and plot; (*c*) reconnoissance of a creek or road by Jacob's staff or compass; (*d*) levelling for profile; (*e*) ploting with plane table.

Parties of about six each would go out into the country and each do the problem assigned. If possible, on the same day instruments would be exchanged; bench marks located; each party would check up another's work, and differences in results explained if possible. It might be that a third party would have to go over the same problem. If the problem were too long for a part of a day then the whole day was taken. There was the utmost enthusiasm. After all problems were done, then came the actual plotting of them in a careful and thorough way.

In those days Washington had a strong algebra course in the eighth grade. Then, as now, geometry was always taught from the reasoning standpoint and hundreds of original exercises worked during each year. Fourth-year pupils easily completed, twenty years ago, the Cornell requirements in advanced algebra which were far in excess of the present ones, and in

addition studied conics, covering the straight line and circle exhaustively and the other conics comparatively, each pupil making a table of analogous formulas.

In those days also Cornell University gave admission credit for spherical trigonometry taken under me by boys desiring to do so upon the condition that these boys should pass the examination set by myself. It is being urged now that spherical trigonometry should be taken up by secondary schools coupled with the elements of navigation as an addenda to solid geometry; not very extensive to be sure, but enough for the boy to get the idea preliminary to college work. Is it not evident that basic ideas and principles persist and live?

Again, many years ago emphasis was placed on oral analysis or mental arithmetic, to develop the ability to think quickly and accurately, to compute mentally and to express one's thoughts orally in clear concise language. And this is what the army officers are pleading for now.

It would seem then that the main tendencies in secondary schools at the present time because of the war and the armistice, should, if the expectations of the officers with whom I have talked recently are to be realized, be merely to restore the very conditions which existed years ago and to add thereto some practical applications which had come into vogue some years before our declaration of war. In other words it would seem that those trained in the secondary mathematics long ago or even not so very long ago have stood the test of the crisis brought on by the war fairly well and that our secondary mathematical courses are not in such a bad state nor so barren or useless as our critics would have every one believe. If that which colleges seem to be agreed upon as to courses in navigation is desirable, then secondary schools can help them by placing a course in solid geometry after trigonometry followed by the elements of navigation, provided that colleges do not require that a full half year be devoted exclusively to the solid geometry *per se*.

I am hopeful moreover that high schools and grade schools may so speed up as to gain the two years of advantage which some foreign countries have over us. The time so gained could be used to splendid advantage.

As to Colleges.

This will be considered very briefly.

The issues of the *American Mathematical Monthly* for September, October and November, 1918, have gone deeply into the matter of collegiate mathematics for war service, much of which will doubtless continue to function.

Because of the certainty of a great navy and a great merchant marine, it seems to be the consensus of opinion that a course in navigation should be provided in all colleges, again stressing the need for trigonometry which should be taught in all secondary schools with emphasis on the derivation and use of computation formulae. This course in navigation is set out in great detail conforming more or less to that of the Naval Academy and presupposing a course in spherical trigonometry. Other courses are considered, such as naval ordnance and gunnery, nautical astronomy, firing data, graphical solutions, etc., but are not insisted upon.

The November (1918) number of the *American Mathematical Monthly* contains a brief account of the work of the Mathematical Association of America at its meeting held at Dartmouth College, September 5, 6, 7, 1918, where some fifty professors in various colleges, after almost continuous conference, agreed upon the character and contents of: three twelve-week terms for general military students in college; two twelve-week terms in navigation; one eight-weeks course in descriptive geometry. This report went to the government offices in Washington by way of suggestion and it was expected that the government's directions would be issued immediately thereafter. This expectation was not realized, but it is certain that the report must have some effect upon the colleges. So much for specific observations.

In Conclusion.

We are face to face with a general problem of far-reaching significance. There will begin, in the reconstruction of education, a struggle to have all kinds of subjects included in the curriculum and there will be a prolonged fight to industrialize education. To a very limited extent this is right because of the

great need to reconstruct the material world, but there is a greater need to reconstruct the morals and ideals of the world and its component parts and to prevent materialism from swallowing up idealism. To this end our mathematical teaching should respond to the touchstone of truth for truth's sake. It must be remembered forever that a deadly efficient practical material empire subverting all else to that of material gain and material expression with no desire to have proper ideals, lost that which stood for a soul and plunged the world into a hideous cataclysm of all but universal ruin.

I believe in practical mathematics, but I believe more in a calm study of the eternal truths of mathematics just at present with the hope that it may stabilize the student in this most critical period in life of his country and himself.

Mathematics has more than justified the position it has always maintained as the corner-stone or keystone upon which the wise direction and successful issues of this war have depended. Turn where you will, you must acknowledge that, either immediately or remotely without mathematics and its applications the war could not have been carried on and won. I do not maintain that it had exclusive jurisdiction; there is glory for all. I do maintain that from every standpoint mathematics has demonstrated conclusively its pre-eminent worth and its uncontrovertible right to be included in the curriculum almost from the very beginning to the end of time.

It is needed now as never before, as a study for its own inherent characteristics to steady the mind and soul afloat on the seething cross currents of Bolshevism which are bound to set in in education government and life. It is one of the few subjects incapable of being distorted because of its inherent nature and is ready at all times to render practical aid to others while acting as a potent factor to keep alive the purity of thought and truth.

FRANKLIN SCHOOL,
WASHINGTON, D. C.
November, 1918.

TEACHING PRACTICAL MATHEMATICS EFFICIENTLY.

By Charles H. Sampson.

Practical mathematics is a valuable tool for any tradesman to own. And yet a very small percentage indeed of these men fully realize the value of this tool. *Why?*

There are several reasons, apparently, why there is not the proper conception of the value of practical mathematics as a part of one's equipment. These may be enumerated as follows:

1. Too many tradesmen are satisfied to remain cogs in the industrial machine of which they are a part. A day's work to them means running a machine and what they get for doing it. Nothing more. They are not interested in adding another tool to the kit which seems to them sufficiently complete to enable them to obtain what they are satisfied to obtain. These men belong to the non-ambitious class. Many of them could never appreciate the value of this suggested tool.

2. But there are some mechanics that seem to realize that after all there is something worth thinking about in the suggestion that they devote some time to a study of the subject of practical mathematics. This group is unfortunate in some respects. They know that they need something of this sort but they don't know what they need. They may possibly have no one to advise and guide them.

3. And then there is a third group composed of men who not only fully realize the assistance that the tool—practical mathematics—can be to them but who know, in addition to this, about what they want. These men will be the most progressive in the shop. Many of them are doubtless foreman. They will all be thinkers, more or less clean-cut in appearance; without question the best all-round men in their particular group.

It is this group of men that I wish to consider here. How can they best be taught practical mathematics effectively?

It is quite necessary that the first step be a study of existing conditions. The general subject content of the course as far as its determination is concerned, should be the result of this study. For illustration, it would not be wise to give a class of practical electricians the same problems for solution as would be presented to a class of machinists. The field must be properly surveyed and course content decided after this survey has taken place.

Now this preliminary work for the purpose as indicated does not by any means determine the total content of the course. The fact that many are inclined to think that it does, is responsible for the failure of many classes in this subject. *A great deal of emphasis should be given the fundamental mathematical processes.* I have seen it happen so many times—the assurance on the part of the members of the class that they *do know* how to add and subtract and multiply and divide. But they don't! Men of the age generally enrolled in these classes are liable to say: "I took all of that back in the grammar school and don't need it now." But they fail to fully appreciate what it really means to attempt to resurrect memories of processes usually covered in an unsatisfactory manner several years past. These things are easily forgotten unless constantly used. And they are not usually—constantly used.

So it is exceedingly important that the general content, that is, the problems to be solved, be built upon and around the usual fundamental mathematical truths.

What then should the course consist of? My experience with hundreds of students tells me that the following general outline is the best for all-round use. Note that there are twenty assignments in all. This is a good number to use for two or three sound reasons. Interest in any given year cannot be held for much longer than that; the courses would generally be offered during the late fall, winter and spring and twenty weeks (one assignment a week) is about all the time conveniently available; the subject matter can be very well covered in just about this length of time.

My idea of the assignment subjects is as follows:

Assignment 1.—The Fundamental Operations Applied to whole Numbers.

Assignment 2.—The Fundamental Operations Applied to Fractions.
Assignment 3.—Decimals and their Application.
Assignment 4.—Percentage Practically Applied.
Assignment 5.—The Solution of Simple Equations.
Assignment 6.—Ratio and Proportion.
Assignment 7.—Powers and Roots of Numbers.
Assignment 8.—Solution of Formulas.
Assignment 9.—Graphs Practically Applied.
Assignment 10.—Introduction to Plane Geometry, Rules and Definitions.
Assignment 11.—Mensuration of Four-sided Figures.
Assignment 12.—Mensuration of the Triangle.
Assignment 13.—The Solution of the Right Triangle.
Assignment 14.—The Hexagon and Similar Geometrical Figures.
Assignment 15.—Mensuration of the Circle.
Assignment 16.—Common Geometrical Constructions.
Assignment 17.—Rules and Definitions Relating to Geometrical Solids.
Assignment 18.—Prisms and Cylinders.
Assignment 19.—Pyramids and Cones.
Assignment 20.—The sphere.

This outline may seem to be too academic. But the important thing is to build practical problems around it. Isn't it just as easy to illustrate the use of the formula for finding the volume of a hexagonal prism by applying it to a hexagonal bar of iron or a hexagonal nut as to think of the model as merely a geometrical solid? It certainly is possible and it must be done if the course is to be interesting and profitable to those who take it. Shop problems for machinists, electrical problems for electricians, building problems for carpenters—all requiring for their solution a knowledge of the fundamental truths. Very simple but very important.

The instructor is such an important element in this kind of work. Any instructor positively will not do. The mere fact that one has been a successful teacher of mathematics in a high school or college does not imply in the slightest degree that he can conduct a class in practical mathematics successfully. The

instructor should first of all be able to get down to the level of the men whom he is teaching. He must be able to talk in terms that they can understand; he must be unusually informal in his attitude as he talks although he must at all times command attention and respect. Those who teach practical subjects should at least have some understanding of how things are done in actual practice. This work is not for a dreamer or a theorist. The teacher will be much better qualified to present the subject if he has had some actual shop or field experience himself. He can then more easily appreciate the attitude of the men and can be more certain that he will "make no breaks" as he talks and explains.

Keen interest and desire for worthwhile accomplishment is a reliable asset for the teacher to possess. He who assumes the responsibility of successfully instructing in the subject must be willing to spend some time diagnosing the particular problem in hand. He will usually find an opportunity to apply treatment in the form of special problems to illustrate some point brought out during a discussion or he may find an opportunity to interpose an interesting lecture on some subject of general interest. There is ample opportunity to test one's ingenuity in this kind of work.

Definite lesson arrangements should always be given and these should be completed and passed in when they are due. An attempt should be made to promote habits of neatness by requiring that all work be neatly arranged on the paper and that everything be in ink. The instructor for his part should get the papers (well corrected) back to the students at the next meeting after their receipt and before starting on the advanced lesson he should endeavor to draw out questions on points not understood and explain the mistakes which seem most evident in the papers to be returned.

The "drawing-out questions process" is a rather difficult one to handle satisfactorily. These men do not, generally speaking, like to ask questions. My method has always been to ask the questions on the points which the corrected papers indicate are not entirely clear and direct these questions at the man whom I know does not thoroughly understand what the answer should be. In this way discussion is usually started.

Classes should always begin on schedule time and if it is necessary to close at a stated time the period should end exactly when it should end. But if conditions are such that "on occasion" a few minutes overtime can be used to advantage, advantage should be taken of it. This extra period may often prove to be the most important part of the lesson.

One of the most valuable factors in this work (at least I have found it so) is a "follow-up" of absences. If I had a class to-night for example, and John Jones and Jim Smith were absent, each of these gentlemen would receive to-morrow, a personal letter (not a form letter) from me expressing my note of the absence, the hope that their presence can be arranged for at the next meeting, and the next lesson assignment. With me, this has been an important factor in maintaining the attendance to the desired percentage.

At the close of the course a final examination may or may not be desirable. This will depend upon the grade of men in the class as a whole. Generally speaking, I do not consider it advisable. These men are not used to taking examinations and are liable to become confused. The examination mark will not be a criterion of their general knowledge of the subject.

There should always be a certificate of completion. And nothing cheap is to be considered. A small engraved certificate dignified in appearance seems to be entirely satisfactory. The presentation of these is important. They should not be mailed but exercises should be arranged and a somewhat formal program carried out. This leaves a "good taste" and a desire for future intellectual improvement. If possible, those in authority in the places of employment of the men should be present and they should take part in the exercises.

I am positive that if the above suggestions are followed success in teaching practical mathematics will result. I have seen it accomplished again and again. There is no guess work about it. It is simply the old story of having a task to do and giving the best one has to the doing of it. The right man teaching the right course will do the job and do it right.

HUNTINGTON SCHOOL,
BOSTON, MASS.

NEW BOOKS.

Plane Geometry. By MABEL SYKES and CLARENCE E. COMSTOCK. Chicago, Rand, McNally and Company. Pp. xii + 322.

This book will prove interesting to all teachers who believe that geometry should train the pupils for original thinking. It discards the traditional formal proof to a great extent, giving instead analyses and methods of attack. Outline reviews placing emphasis on the use of propositions are also a good feature.

Elements of Plane Trigonometry, with Brief Tables. By ALFRED MONROE KENYON and LOUIS INGOLD. New York, The Macmillan Company. Pp. xxvii + 129.

This is an elementary text following out in simple form much the same idea that was used by the authors in their Plane and Spherical Trigonometry. It contains all that is necessary for a first course, and emphasizes the parts needed for practical use.

Plane Geometry. By MATILDA AUERBACH and CHARLES BURTON WALSH. Philadelphia, J. B. Lippincott Company. Pp. xvi + 383.

The unique feature of this text is its two part arrangement, the first part being a complete view of plane geometry treated in a manner to emphasize general interest and application, while the second part stresses college preparation.

The amount of material is great, with the advantage of richness, but the probable disadvantage of seeming like a large amount to the pupil. This disadvantage is somewhat emphasized by the arrangement, which gives the impression that the publishers have been too saving of paper.

There is much of excellence in the book, but there is also an occasional point that is open to criticism for looseness. The reviews might be mentioned for their interesting form.

First Book in Algebra. By FLETCHER DURELL and E. E. ARNOLD. New York, Charles E. Merrill Company. Pp. xxxiii + 325.

The subject matter of this book concerns itself less with complications, and more with equations, problems and graphs. It divides each chapter for a first and second reading, so that a simple first course can be followed by a more complete and theoretical second course.

Analysis of problems is emphasized throughout.

Plane Trigonometry. By JOHN WESLEY YOUNG and FRANK MILLETT MORGAN. New York, The Macmillan Company. Pp. vii + 122.

The trigonometrical part of "Elementary Mathematical Analysis" by the same authors has been revised and published in this form.

It emphasizes the numerical side of the subject, giving a minimum of theory. While the work is in condensed form, it seems complete and usable.

Our Little Czecho-Slovak Cousin. By CLARA VOSTROVSKY WINLOW. Boston, The Page Company. Pp. viii + 124.

This is a charming one of the "Little Cousin Series." It combines information and literary value with real interest.

Applied Arithmetic, Books 2 and 3. By N. J. LENNES and FRANCES JENKINS. Philadelphia, J. B. Lippincott Company. Book 2—Pp. ix + 294. (Fifth and Sixth Grades.) Book 3—Pp. ix + 340. (Seventh and Eighth Grades.)

The material in these books is well chosen from the standpoint of interest, ease of use, and usefulness. Topics of doubtful value have been put in the list of supplementary topics.

They are worthy of consideration by any school organized on the eight grade basis.

Fundamentals of High School Mathematics. By HAROLD O. RUGG and JOHN R. CLARK. Yonkers-on-Hudson, N. Y., World Book Company. Pp. xv + 366. Price $1.60.

This ninth year course is perhaps the nearest approach to the recommendations in the preliminary report of the National Committee of any book yet written. It embodies the results of much educational research, and has been subjected to the criticisms of a large body of teachers drawn from every state in the Union.

It merits careful examination by any school that wishes to keep abreast of the latest developments in the pedagogy of algebra.

Applied Mathematics. By EUGENE H. BARKER. Boston, Allyn and Bacon. Pp. viii + 247. Price $1.25.

This book treats of mathematics in business transactions, and applications to common uses, such as those of the household, the farm and the office. It presents the material well, and seems suited to such a course in either the junior or senior high school.

Household Arithmetic. By KATHARINE F. BALL and MIRIAM E. WEST. Philadelphia, J. B. Lippincott Company. Pp. 271.

A very practical and complete book for girls. It is well planned and should prove a valuable preparation for efficient management of a home.

The book is organized under the headings—Budgets and Accounts, Shelter, Operation, Clothing, Food, and Higher Life.

Report on the Physical Conditions of the Elementary Public Schools of Baltimore City. By the Council of the Allied Association of Public School Teachers of Baltimore—WILLIAM H. MALTBIE, Advising Director and Editor. Pp. 278; price 25 cents.

This survey of physical conditions is one of the most complete ever made in any system. It takes up each school in detail, pointing out its needs and giving the complete statistics regarding it.

The report is not only valuable for its information regarding Baltimore, but it may well serve as a model for such work in other cities.

It was made with the coöperation of teachers and school authorities for the purpose of acquainting the public with the conditions that needed correction.